

Thermodynamical Laws in Hořava-Lifshitz Gravity

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In this work, we have investigated the validity of GSL of thermodynamics in a universe (open, closed and flat) governed by Hořava-Lifshitz gravity. If the universe contains barotropic fluid the corresponding solutions have been obtained. The validity of GSL have been examined by two approaches: (i) robust approach and (ii) effective approach. In robust approach, we have considered the universe contains only matter fluid and the effect of the gravitational sector of HL gravity was incorporated through the modified black hole entropy on the horizon. Effective approach is that all extra information of HL gravity into an effective dark energy fluid and so we consider the universe contains matter fluid plus this effective fluid. This approach is essentially same as the Einstein's gravity theory. The general prescription for validity of GSL have been discussed. Graphically we have shown that the GSL may be satisfied for open, closed and flat universe on the different horizons with different conditions.

I. INTRODUCTION

Recently, a power-counting renormalizable theory of gravity was proposed by Hořava [1-4]. This is a non-relativistic theory of gravity and is expected to recover Einstein's general relativity at large scales. This theory does not have the full diffeomorphism invariance. Although presenting an infrared (IR) fixed point, namely General Relativity, in the UV the theory exhibits an Lifshitz type anisotropic scaling between time and scale, so it is commonly known as Hořava-Lifshitz (HL) gravity. The HL gravity theory has attracted much attention as a candidate quantum field theory of gravity with $z = 3$ in the UV, where z measures the degree of anisotropy between space and time. In 3+1 dimensions, the Hořava-Lifshitz theory has a $z = 3$ fixed point in the UV and flows to a $z = 1$ fixed point in the IR, which is just the classical Einstein-Hilbert gravity theory. In this theory the effective coupling constant is dimensionless. This theory of gravity has four possible versions so far - with/without the detailed balance condition and with/without projectability condition. Among these version without detailed balance and with the projectability condition is the most viable one.

Hořava-Lifshitz gravity has been studied and extended in detail [5] and it has been applied as the cosmological framework of the universe [6, 7]. It was extensively applied to a resolution of the cosmological problem including inflation and non-Gaussianity in [8, 9], and new solutions were constructed in [10, 11]. Amongst the very interesting physical implications are the novel solution subclass [10,12], the gravitational wave production [13], dark energy phenomenology [14], astrophysical phenomenology [15], observational constraints [16] etc. Static spherically symmetric black hole solutions in Hořava-Lifshitz gravity has been studied in [10]. However, despite this extended research, there are still many ambiguities if Hořava-Lifshitz gravity is reliable and capable of a successful description of the gravitational background of our world, as well as of the cosmological

behavior of the universe [17]. There are several work on the thermodynamical properties of black hole in Hořava-Lifshitz gravity theory [18].

Here we consider the universe is treated as a thermodynamical system and we discuss the GSL of thermodynamics in Hořava-Lifshitz gravity theory. Till now there are some works [19] have been done on thermodynamics in FRW model with HL gravity. Here we try to examine the validity of GSL on the horizons in Hořava-Lifshitz gravity theory. For this purpose, we investigate two approaches. The first approach is to consider the FRW universe is filled with only matter field and using first law, we examine the validity of GSL on the horizons and give some conditions in HL gravity. Second approach is to absorb all the extra terms of HL gravity in an dark energy component and we consider the universe is filled with dark matter and this type of dark energy. But the first approach is more interesting, because we have directly get the additional information of HL gravity model. Recently Jamil et al [20] have discussed the validity of GSL considering these two types of approaches. We have extend their work in different horizons like, Hubble, apparent, particle and event horizons in general manner. Finally, we give some solutions for barotropic fluid and justify the validity of GSL in different horizons.

II. HOŘAVA-LIFSHITZ GRAVITY THEORY

We briefly review here the scenario where the cosmological evolution is governed by Hořava-Lifshitz gravity [6, 7]. The dynamical variables are the lapse rate and shift functions, N and N_i respectively, and the spatial metric g_{ij} . In the (3+1) dimensional Arnowitt-Deser-Misner formalism the full metric [21] is written as

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad (1)$$

The scaling transformation of the coordinates reads: $t \rightarrow l^3 t$ and $x^i \rightarrow l x^i$. The gravitational action is decomposed into a kinetic and a potential part as $S = \int dt d^3x \sqrt{g} N (\mathcal{L}_K + \mathcal{L}_V)$. Under the detailed balance con-

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dition the full action condition of Hořava-Lifshitz gravity is given by

$$S = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2}{2\omega^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu \epsilon^{ijk}}{2\omega^2 \sqrt{g}} R_{il} \nabla_j R_k^l + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \right] \quad (2)$$

where

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) \quad (3)$$

is the extrinsic curvature and

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k (R_i^j - \frac{1}{4} R \delta_l^j) \quad (4)$$

is known as Cotton tensor and the covariant derivatives are defined with respect to the spatial metric g_{ij} . ϵ^{ijk} is the totally antisymmetric unit tensor, λ is a dimensionless coupling constant and the variable κ , ω and μ are constants with mass dimensions -1 , 0 , 1 respectively. Also Λ is a positive constant, which as usual is related to the cosmological constant in the IR limit. In order to incorporate the (dark plus baryonic) matter component one adds a cosmological stress-energy tensor to the gravitational field equations, by demanding to recover the usual general relativity formulation in the low energy limit. Let us suppose the energy density and pressure are denoted by ρ and p respectively.

Now, in order to focus on cosmological frameworks, we impose the so called projectability condition [17] and use a Friedmann-Robertson-Walker (FRW) metric,

$$N = 1, g_{ij} = a^2(t) \gamma_{ij}, N^i = 0 \quad (5)$$

with

$$\gamma_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2, \quad (6)$$

where $k = 0, -1, +1$ corresponding to flat, open and closed respectively. By varying N and g_{ij} , we obtain the non-vanishing equations of motions:

$$H^2 = \frac{\kappa^2}{6(3\lambda - 1)} \rho + \frac{\kappa^2}{6(3\lambda - 1)} \left[\frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} - \frac{\kappa^4 \mu^2 k \Lambda}{8(3\lambda - 1)^2 a^2} \right] \quad (7)$$

and

$$\dot{H} + \frac{3}{2} H^2 = -\frac{\kappa^2}{4(3\lambda - 1)} p - \frac{\kappa^2}{4(3\lambda - 1)} \left[\frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} \right]$$

$$- \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \Big] - \frac{\kappa^4 \mu^2 k \Lambda}{16(3\lambda - 1)^2 a^2} \quad (8)$$

where $H \equiv \frac{\dot{a}}{a}$ is the Hubble parameter. The term proportional to a^{-4} is the usual “dark radiation”, present in Hořava-Lifshitz cosmology while the constant term is just the explicit cosmological constant. For $k = 0$, there is no contribution from the higher order derivative terms in the action. However for $k \neq 0$, there higher derivative terms are significant for small volume i.e., for small a and become insignificant for large a , where it agrees with general relativity. As a last step, requiring these expressions to coincide the standard Friedmann equations, in units where $c = 1$ we set [6, 7],

$$G_c = \frac{\kappa^2}{16\pi(3\lambda - 1)} \quad (9)$$

$$\frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2} = 1 \quad (10)$$

where G_c is the “cosmological” Newton’s constant. We mention that in theories with Lorentz invariance breaking (such is Hořava-Lifshitz one) the “gravitational” Newton’s constant G , that is the one that is present in the gravitational action, does not coincide with G_c , that is the one that is present in Friedmann equations, unless Lorentz invariance is restored [22], where

$$G = \frac{\kappa^2}{32\pi} \quad (11)$$

as it can be straightforwardly read from the action (2). In the IR ($\lambda = 1$) where Lorentz invariance is restored, $G_c = G$. Using the above identifications, we can re-write the Friedmann equations (7) and (8) as,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_c}{3} \rho + \frac{k^2}{2\Lambda a^4} + \frac{\Lambda}{2} \quad (12)$$

and

$$\dot{H} + \frac{3}{2} H^2 + \frac{k}{2a^2} = -4\pi G_c p - \frac{k^2}{4\Lambda a^4} + \frac{3\Lambda}{4} \quad (13)$$

If we consider the matter is conserved then the continuity equation is given by

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (14)$$

In the next section, we will describe the general condition for validity of GSL of thermodynamics on the Hubble, apparent, particle and event horizons due to presence of HL gravity in FRW universe.

III. GENERALIZED SECOND LAW OF THERMODYNAMICS IN FRW UNIVERSE IN HOŘAVA-LIFSHITZ GRAVITY

We consider the FRW universe in Hořava-Lifshitz gravity as a thermodynamical system with the horizon surface

as a boundary of the system. To study the generalized second law (GSL) of thermodynamics through the universe we deduce the expression for normal entropy using the Gibb's equation of thermodynamics [27]

$$T_X dS_{IX} = p dV + d(E_X) \quad (15)$$

where, S_{IX} , p , V and E_X are respectively entropy, pressure, volume and internal energy within the Hubble/apparent/particle/event horizon and T_X is the temperature on the Hubble horizon ($X = H$)/apparent horizon ($X = A$)/particle horizon ($X = P$)/event horizon ($X = E$). Here the expression for internal energy can be written as $E_X = \rho V$. Now the volume of the sphere is $V = \frac{4}{3}\pi R_X^3$, where R_X is the radius of the Hubble horizon (R_H)/apparent horizon (R_A)/particle horizon (R_P)/event horizon (R_E) defined by [27, 34]

$$R_H = \frac{1}{H}, \quad (16)$$

$$R_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}} = \frac{1}{\sqrt{H^2 + k(1+z)^2}}, \quad (17)$$

$$R_P = a \int_0^t \frac{dt}{a} = a \int_0^a \frac{da}{a^2 H} = \frac{1}{1+z} \int_z^\infty \frac{dz}{H} \quad (18)$$

and

$$R_E = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{a^2 H} = \frac{1}{1+z} \int_{-1}^z \frac{dz}{H} \quad (19)$$

where z is the redshift defined by $z = \frac{1}{a} - 1$. Now differentiating (16) - (19), with respect to time t , we find

$$\dot{R}_H = -\frac{\dot{H}}{H^2} \quad (20)$$

$$\dot{R}_A = H R_A^3 (k(1+z)^2 - \dot{H}) \quad (21)$$

$$\dot{R}_P = H R_P + 1 \quad (22)$$

and

$$\dot{R}_E = H R_E - 1 \quad (23)$$

The temperature on the Hubble/ apparent/ particle/ event horizon is chosen as

$$T_X = \frac{1}{2\pi R_X} \quad (24)$$

Now from (15) we obtain, the rate of change of internal entropy (using (15) and (24)) as,

$$\dot{S}_{IX} = 8\pi^2 R_X^3 (\rho + p) (\dot{R}_X - H R_X) \quad (25)$$

In case of black holes in Hořava-Lifshitz gravity and under the detailed balance condition, the expression for the entropy of the horizon [11, 23] is given by

$$S_X = \frac{4\pi^2 \kappa^2 \mu^2}{4} [\Lambda R_X^2 + 2k \ln(\sqrt{\Lambda} R_X)] \quad (26)$$

which implies (after differentiating)

$$\dot{S}_X = \frac{2\pi}{G} R_X \dot{R}_X + \frac{2\pi k}{G \Lambda R_X} \dot{R}_X \quad (27)$$

where in IR limit ($\lambda = 1$) we get $G_c = G$.

Therefore rate of change of total entropy in HL gravity is obtained as (adding (25) and (27))

$$\begin{aligned} \dot{S}_{IX} + \dot{S}_X &= 8\pi^2 R_X^3 (\dot{R}_X - H R_X) (\rho + p) \\ &+ \frac{2\pi}{G} \left(R_X + \frac{k}{\Lambda R_X} \right) \dot{R}_X \end{aligned} \quad (28)$$

The GSL in HL gravity will satisfy if the following condition holds:

$$\begin{aligned} \dot{S}_{IX} + \dot{S}_X &\geq 0 \quad i.e., \quad 8\pi^2 R_X^3 (\dot{R}_X - H R_X) (\rho + p) \\ &+ \frac{2\pi}{G} \left(R_X + \frac{k}{\Lambda R_X} \right) \dot{R}_X \geq 0 \end{aligned} \quad (29)$$

From the above restrictions, we can not draw any definite conclusions for validity of GSL in HL gravity for flat, open or closed FRW universe on different horizons. On the apparent horizon, Jamil et al [20] have found that for flat and closed universe, GSL is always satisfied but for open universe, the GSL may be satisfied for some conditions upon Λ . For $k = 0$ we get back similar to the simple Einstein's gravity. So we are interested to get results only for $k \neq 0$. In this case, the results for HL gravity will be obtained. In the next section, we'll consider the matter fluid is followed by barotropic equation of state. For this type of fluid, we'll find some solutions and it is easy to verify the validity of GSL on different horizons.

IV. VALIDITY OF GSL ON HUBBLE, APPARENT, PARTICLE AND EVENT HORIZONS IN THE PRESENCE OF BAROTROPIC FLUID

Let us consider the equation of state for the barotropic fluid is

$$p = w\rho \quad (30)$$

where w is a constant. Now solving equation (14) we get the expression for energy density ρ in terms of redshift z as

$$\rho = \rho_0(1+z)^{3(1+w)} \quad (31)$$

where ρ_0 is integration constant. Solving (12) and (13) for H and \dot{H} in terms of redshift z , obtained as

$$H = \left[\frac{8\pi G_c}{3} \rho_0(1+z)^{3(1+w)} + \frac{k^2}{2\Lambda}(1+z)^4 + \frac{\Lambda}{2} - k(1+z)^2 \right]^{\frac{1}{2}} \quad (32)$$

and

$$\dot{H} = -4\pi G_c \rho_0(1+w)(1+z)^{3(1+w)} - \frac{k^2}{\Lambda}(1+z)^4 + k(1+z)^2 \quad (33)$$

Also the radii of particle and event horizons (from (18) and (19)) can be written as

$$R_P = \frac{1}{1+z} \int_z^\infty \left[\frac{8\pi G_c}{3} \rho_0(1+z)^{3(1+w_M)} + \frac{k^2}{2\Lambda}(1+z)^4 + \frac{\Lambda}{2} - k(1+z)^2 \right]^{-\frac{1}{2}} dz \quad (34)$$

and

$$R_E = \frac{1}{1+z} \int_{-1}^z \left[\frac{8\pi G_c}{3} \rho_0(1+z)^{3(1+w_M)} + \frac{k^2}{2\Lambda}(1+z)^4 + \frac{\Lambda}{2} - k(1+z)^2 \right]^{-\frac{1}{2}} dz \quad (35)$$

Let us now proceed to calculation of total entropy variation with respect to time t .

Using equations (20) and (28), the total entropy variation on Hubble horizon is obtained as

$$\begin{aligned} \dot{S}_{IH} + \dot{S}_H &= \frac{2\pi}{GH^3} \left(\frac{\dot{H}}{H^2} + 1 \right) \left[\dot{H} - k(1+z)^2 \right. \\ &\quad \left. + \frac{k^2(1+z)^4}{\Lambda} \right] - \frac{2\pi\dot{H}}{GH^2} \left(\frac{1}{H} + \frac{kH}{\Lambda} \right) \end{aligned} \quad (36)$$

Using equations (21) and (28), the total entropy variation on Apparent horizon is obtained as

$$\begin{aligned} \dot{S}_{IA} + \dot{S}_A &= \left[\frac{2\pi H R_A^6}{G} \left(\dot{H} + k(1+z)^2 + \frac{k^2(1+z)^4}{\Lambda} \right) \right. \\ &\quad \left. - \frac{2\pi k}{G\Lambda} R_A^2 H \right] \times \left(\dot{H} - k(1+z)^2 \right) + \frac{2\pi k^2}{G\Lambda} R_A^4 H (1+z)^4 \end{aligned} \quad (37)$$

Using equations (22) and (28), the total entropy variation on particle horizon is obtained as

$$\dot{S}_{IP} + \dot{S}_P = -\frac{2\pi R_P^3}{G} \left(\dot{H} - k(1+z)^2 + \frac{k^2(1+z)^4}{\Lambda} \right)$$

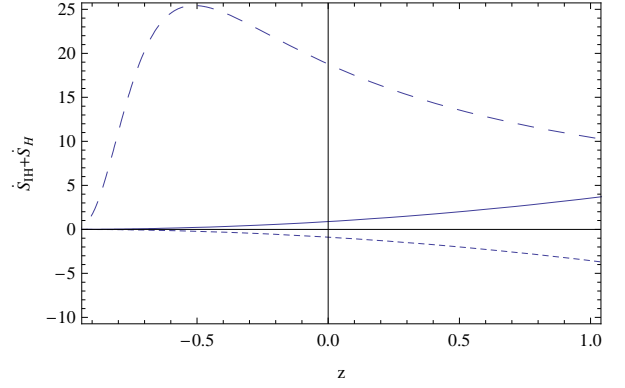


Fig.1

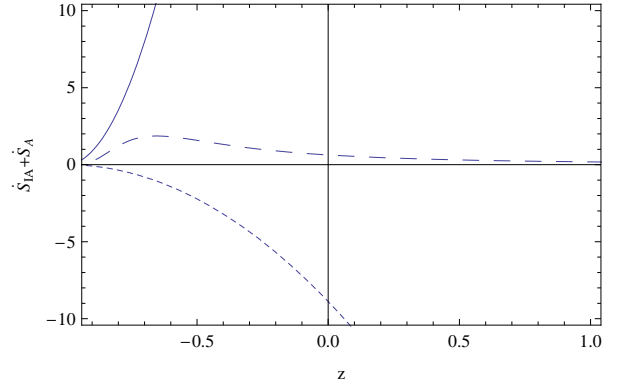


Fig.2

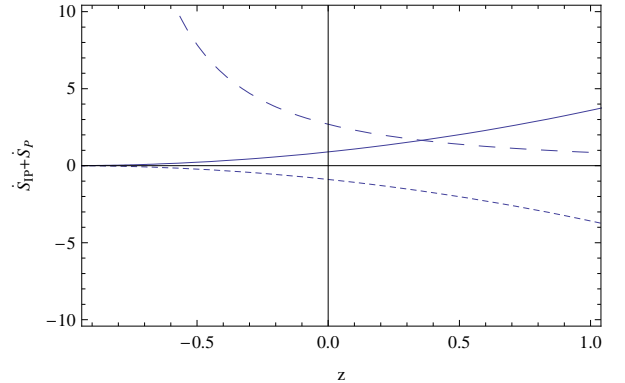


Fig.3

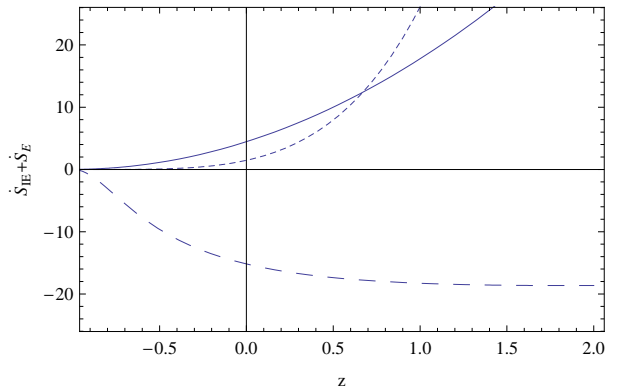


Fig.4

Figs. 1, 2, 3 and 4 represent respectively the variations of $(\dot{S}_{IH} + \dot{S}_H)$, $(\dot{S}_{IA} + \dot{S}_A)$, $(\dot{S}_{IP} + \dot{S}_P)$ and $(\dot{S}_{IE} + \dot{S}_E)$ against redshift z for $w = -1/3$ and $k = 0, \pm 1$. The dashed line.

$$+ \frac{2\pi}{G} \left(R_P + \frac{k}{\Lambda R_P} \right) (H R_P + 1) \quad (38)$$

Using equations (23) and (28), the total entropy variation on event horizon is obtained as

$$\begin{aligned} \dot{S}_{IE} + \dot{S}_E &= \frac{2\pi R_E^3}{G} \left(\dot{H} - k(1+z)^2 + \frac{k^2(1+z)^4}{\Lambda} \right) \\ &+ \frac{2\pi}{G} \left(R_E + \frac{k}{\Lambda R_E} \right) (H R_E - 1) \quad (39) \end{aligned}$$

The time variations to total entropies on the Hubble, apparent, particle and event horizons have been drawn against z in figures 1 - 4 respectively for $k = 0, \pm 1$. From graphical representations we make the following conclusions:

(a) In figure 1, we see that $(\dot{S}_{IH} + \dot{S}_H)$ is (i) always positive for $k = +1$, (ii) always negative for $k = -1$ (iii) is positive upto certain stage and may be negative at late stage for $k = 0$. So on Hubble horizon, the GSL is satisfied always for closed universe and for open universe, GSL breaks down. Also for flat universe, GSL may be satisfied on Hubble horizon but at late stage ($z < -0.8$) GSL breaks down.

(b) In figure 2, we see that $(\dot{S}_{IA} + \dot{S}_A)$ is (i) always positive for $k = 0$ and $+1$, (ii) is negative for $k = 0$. So on apparent horizon, the GSL is always satisfied for closed and flat universe. Also for open universe, GSL breaks down. The result coincides with the work of Jamil et al [20].

(c) In figure 3, we see that $(\dot{S}_{IP} + \dot{S}_P)$ is (i) always positive for $k = 0$ and $+1$ and (ii) always negative for $k = -1$. So on particle horizon, the GSL is satisfied always for closed and flat universe and for open universe, GSL breaks down.

(d) In figure 4, we see that $(\dot{S}_{IE} + \dot{S}_E)$ is (i) always positive for $k = -1$ and $+1$ and (ii) always negative for $k = 0$. So on event horizon, the GSL is satisfied always for closed and open universe and for flat universe, GSL breaks down.

The above conclusions are valid in HL gravity theory with barotropic fluid solutions ($w = -1/3$). For other types of solutions, we may obtain similar types of results. So, in HL gravity, the GSL may be satisfied on different horizons.

V. GENERALIZED SECOND LAW OF THERMODYNAMICS : AN EFFECTIVE APPROACH

In the effective approach to HL gravity theory, we assume that the universe contains the matter fluid and dark energy fluid. So the problem is equivalent to the Einstein's gravity with two fluids. It can be introduced an effective dark energy defining the energy density ρ_D and pressure p_D in the Friedmann equations (7) and (8) as

$$\rho_D \equiv \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \quad (40)$$

and

$$p_D \equiv \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \quad (41)$$

which after the identification (9) and (10), can be written as,

$$\rho_D \equiv \frac{1}{16\pi G_c} \left(\frac{3k^2}{\Lambda a^4} + 3\Lambda \right) \quad (42)$$

and

$$p_D \equiv \frac{1}{16\pi G_c} \left(\frac{k^2}{\Lambda a^4} - 3\Lambda \right) \quad (43)$$

If we assume the matter fluid is conserved then equation (14) holds and in this case the dark energy conservation equation will be

$$\dot{\rho}_D + 3H(\rho_D + p_D) = 0 \quad (44)$$

Therefore, the Friedmann equations (12) and (13) turn into the forms

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_c}{3}(\rho + \rho_D) \quad (45)$$

and

$$\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_c(p + p_D) \quad (46)$$

As in the previous section, we find the variations of entropies for matter fluid and dark energy fluid respectively as

$$\dot{S} = \frac{4\pi}{T_X}(\rho + p)R_X^2(\dot{R}_X - H R_X) \quad (47)$$

and

$$\dot{S}_D = \frac{4\pi}{T_X}(\rho_D + p_D)R_X^2(\dot{R}_X - H R_X) \quad (48)$$

Since the effective gravitational sector is now the standard general relativity, the horizon entropy will be given by

$$S_X = \frac{\pi R_X^2}{G} \quad (49)$$

where IR limit ($\lambda = 1$) which allows us to simplify $G_c = G$. Therefore, taking derivative, we get

$$\dot{S}_X = \frac{2\pi R_X \dot{R}_X}{G} \quad (50)$$

Adding the relations (47), (48) and (50), the rate of change of total entropy is obtained as

$$\dot{S}_{tX} = \dot{S}_D + \dot{S} + \dot{S}_X = 8\pi^2 R_X^3 (\dot{R}_X - HR_X)(\rho + p + \rho_D + p_D) + \frac{2\pi R_X \dot{R}_X}{G} \quad (51)$$

From the field equations (45) and (56) we get

$$\rho + p + \rho_D + p_D = -\frac{1}{4\pi G_c} \left(\dot{H} - \frac{k}{a^2} \right) \quad (52)$$

Substituting expressions for different horizons with radius R_X in (51) we get the following results:

For Hubble horizon,

$$\dot{S}_{tH} = \frac{2\pi}{GH^3} \left(\frac{\dot{H}}{H^2} + 1 \right) \left(\dot{H} - \frac{k}{a^2} \right) - \frac{2\pi\dot{H}}{GH^3} \quad (53)$$

For apparent Horizon,

$$\dot{S}_{tA} = \frac{2\pi}{G} R_A^6 H \left(\dot{H} - \frac{k}{a^2} \right)^2 \quad (54)$$

For particle horizon,

$$\dot{S}_{tP} = -\frac{2\pi R_P^3}{G} \left(\dot{H} - \frac{k}{a^2} \right) + \frac{2\pi R_P}{G} (HR_P + 1) \quad (55)$$

For event horizon,

$$\dot{S}_{tE} = \frac{2\pi R_E^3}{G} \left(\dot{H} - \frac{k}{a^2} \right) + \frac{2\pi R_E}{G} (HR_E - 1) \quad (56)$$

From (53), we see that $\dot{S}_{tA} \geq 0$ always for $k = 0, \pm 1$. So GSL is satisfied on apparent horizon for flat, open and closed FRW universe in Hořava-Lifshitz gravity. But \dot{S}_{tH} , \dot{S}_{tP} and \dot{S}_{tE} will be positive if the r.h.s. of equations (53), (55) and (56) are positive. So validity of GSL on Hubble, particle and event horizons depend on the values of H , \dot{H} and horizons radii for flat, open and closed FRW universe in Hořava-Lifshitz gravity. The expressions of H , \dot{H} , R_H , R_P and R_E for barotropic fluid model with HL dark energy are given in equations (16) and (32) - (35). The time variations of total entropies on Hubble, particle and event horizons are presented in figures 5 - 7 for $k = 0, \pm 1$. From graphical representations we make the following conclusions:

(a) In figure 5, we see that \dot{S}_{tH} is (i) always positive for $k = -1$ and $+1$, (ii) is positive upto certain stage and may be negative at late stage for $k = 0$. So on Hubble horizon, the GSL is satisfied always for open and closed universe. Also for flat universe, GSL may be satisfied on Hubble horizon but at late stage ($z < -0.8$) GSL breaks down.

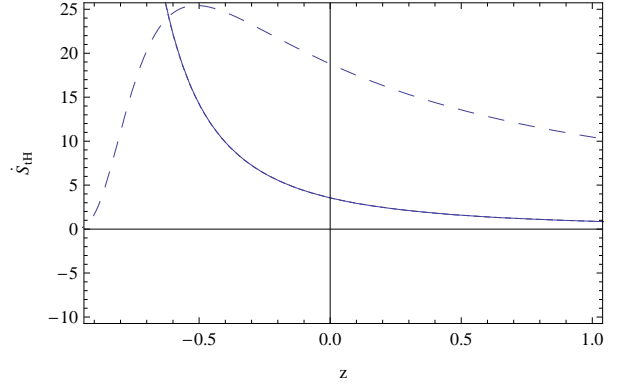


Fig.5

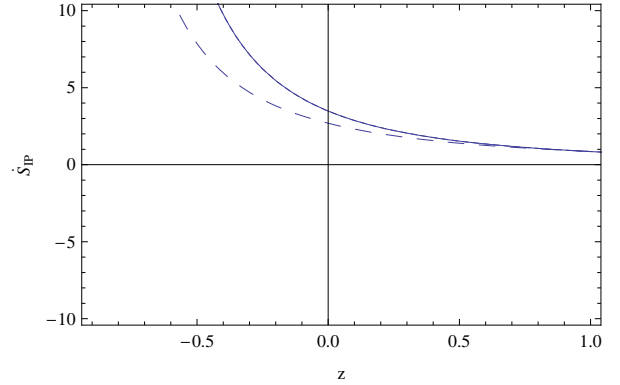


Fig.6

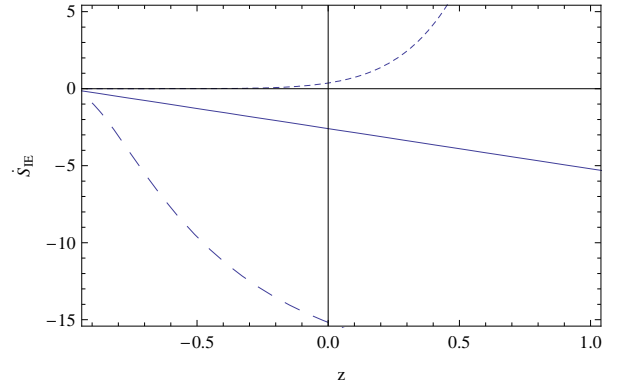


Fig.7

Figs. 5, 6 and 7 represent respectively the variations of \dot{S}_{tH} , \dot{S}_{tP} and \dot{S}_{tE} against redshift z for $w = -1/3$ and $k = 0, \pm 1$.

The dashed line, dotted line and filled line represent for $k = 0, -1$ and $+1$ respectively.

(b) In figure 6, we see that \dot{S}_{tP} is always positive for $k = 0, \pm 1$ and $+1$. So on particle horizon, the GSL is satisfied always for open, closed and flat universe.

(c) In figure 7, we see that \dot{S}_{tE} is always negative for $k = 0, \pm 1$. So on event horizon, the GSL cannot be satisfied for open, closed and flat universe.

The above conclusions are valid in HL gravity theory with barotropic fluid solutions ($w = -1/3$). For other types of solutions, we may obtain similar types of results. So, in HL gravity, the GSL may or may not be satisfied

on different horizons.

VI. DISCUSSIONS

In this work, we have investigated the validity of GSL of thermodynamics in a universe (open, closed and flat) governed by Hořava-Lifshitz gravity. If the universe contains barotropic fluid the corresponding solutions have been obtained. Consider the universe as a thermodynamical system bounded by horizons. The validity of GSL have been examined by two approaches: (i) robust approach and (ii) effective approach. In robust approach, we have considered the universe contains only matter fluid and the effect of the gravitational sector of HL gravity was incorporated through the modified black hole entropy on the horizon. The general prescription for validity of GSL have been discussed. But we cannot draw any definite conclusion for validity of GSL in open, closed and flat models. So graphical experiments have been investigated for final conclusion. The total variations of entropies on Hubble, apparent, particle and event horizons have been presented in figures 1 - 4. These figures show that on Hubble horizon, the GSL is satisfied always for closed universe and for open universe, GSL breaks down. Also for flat universe, GSL may be satisfied on Hubble horizon but at late stage ($z < -0.8$) GSL breaks down. On apparent horizon, the GSL is always satisfied for closed and flat universe. Also for open universe, GSL breaks down. On particle horizon, the GSL is satisfied always for closed and flat universe and for open universe, GSL breaks down. On event horizon, the GSL is satisfied always for closed and open universe and for flat universe, GSL breaks down.

Effective approach is that all extra information of HL gravity into an effective dark energy fluid and so we consider the universe contains matter fluid plus this effective fluid. This approach is essentially same as the Einstein's gravity theory. In this situation, we have obtained the general conditions for validity of GSL in open, closed and flat model of the universe. It has been seen that on the apparent horizon, the GSL is always valid. But for other horizons, it may or may not be valid. The total variations of entropies on Hubble, particle and event horizons have been presented in figures 5 - 7. From figures, we see that on Hubble horizon, the GSL is satisfied always for open and closed universe. Also for flat universe, GSL may be satisfied on Hubble horizon but at late stage ($z < -0.8$) GSL breaks down. On particle horizon, the GSL is satisfied always for open, closed and flat universe. On event horizon, the GSL cannot be satisfied for open, closed and flat universe.

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